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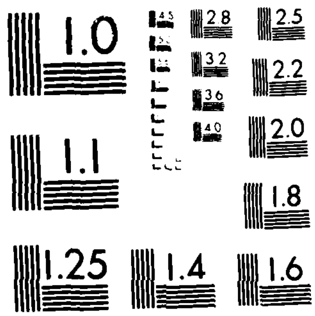
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A UNIFIED METHOD FOR CONSTRUCTING PBIB DESIGNS BASED ON TRIANGULAR AND L_2 SCHEMES

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ABSTRACT

A general and simple method for constructing triangular and L_2 type partially balanced imcomplete block (PBIB) designs is presented. This method unifies several well-known methods and produces many new series of designs. The idea is based on a well-known relation between triangular and L_2 type PBIB designs and line graphs. Triangular or L_2 type PBIB designs can often be obtained by suitably "symmetrizing" a given initial block.

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A UNIFIED METHOD FOR CONSTRUCTING PBIB DESIGNS BASED ON TRIANGULAR AND L_2 SCHEMES

1. Introduction

Triangular and L_2 type partially balanced imcomplete block (PBIB) designs are two important kinds of PBIB designs. Comprehensive tables and a review of some construction methods can be found in Clatworthy (1973). The main purpose of this paper is to present a general and simple method of construction which unifies many of the well-known results scattered in the literature. Each of them becomes a special application of our method which stands at a higher level of abstraction and can be used to produce many new series of designs. The method is simple and enables the construction of such designs of any block size. The idea is based on a well-known relation of triangular and L_2 type PBIB designs to the so-called line graphs in graph theory. For convenience of later reference, we shall review some terminology in graph theory.

Only simple graphs, i.e., those in which there is at most one edge between any two vertices, will be considered in this paper. Two vertices are said to be adjacent if there is an edge between them. If two edges have a common vertex, then they are also called adjacent. The complement of a graph G is defined to be the graph with the same vertices as G in which two vertices are adjacent if and only

if they are not adjacent in G. A <u>complete graph</u> K_n is a graph on n vertices such that any two vertices are adjacent. If the vertices can be divided into two disjoint groups of sizes m_1 and m_2 respectively such that two vertices are adjacent if and only if they are in different groups, then we have a <u>complete</u> (m_1, m_2)-<u>bipartite graph</u>. Such a graph is denoted by K_{m_1, m_2}. For convenience these two groups of vertices will be called the two partite sets of the graph. The <u>line graph</u> $L(G)$ of a simple graph G is defined to be the graph with the edges of G as its vertices; two vertices in $L(G)$ are adjacent if and only if the corresponding edges in G have a common vertex.

A <u>triangular scheme</u> on $v = n(n-1)/2$ varieties is defined by putting the v varieties on the upper-diagonal (and symmetrically on the lower diagonal) of an $n \times n$ square array with the diagonal entries left blank. Two varieties are first associates if they are in the same row or column; otherwise, they are second associates. Considering each variety as a vertex, we can construct a graph on $n(n-1)/2$ vertices in which two vertices are adjacent if and only if the two corresponding varieties are first associates. It is not hard to see that the resulting graph is the line graph of K_n. The variety put in the $(i,j)^{th}$ position of the $n \times n$ square array defining the triangular scheme can be identified with the edge joining the i^{th} vertex and the j^{th} vertex of K_n, $1 < i < j < n$. There are $n(n-1)/2$ edges in K_n, and thus there are $n(n-1)/2$ varieties in a triangular scheme.

Similarly, an L_2 type scheme on $v = n^2$ varieties can be thought of as the line graph of $K_{n,n}$. Recall that an L_2 type scheme is defined by putting the n^2 varieties on an $n \times n$ square array such that two varieties are first associates if and only if they lie in the same row or same column. Now the variety put on the $(i,j)^{th}$ position of the $n \times n$ square array can be identified with the edge in $K_{n,n}$ joining the ith vertex of the first partite set and the jth vertex of the second partite set, $1 < i , j < n$.

The above observations motivate the following definition:

Definition. Let G be a graph with v edges. Then $L(G)$ has v vertices. We say that a binary incomplete block design d with v varieties is a design based on the line graph of G if the following are satisfied:

(i) If vertices i and j of $L(G)$ are adjacent then varieties i and j appear together in λ_1 blocks of d.

(ii) If vertices i and j of $L(G)$ are not adjacent then varieties i and j appear together in λ_2 blocks of d.

According to this definition, a triangular PBIB design with $v = n(n-1)/2$ varieties is a design based on the line graph of K_n, and an L_2 type PBIB design with $v = n^2$ varieties is a design based on the line graph of $K_{n,n}$. In Section 2, we shall present a general method for constructing

designs based on line graphs. Various applications lead to many well-known methods for constructing triangular and L_2 type PBIB designs. These methods are indeed of the same spirit. This will be discussed in Section 3. Some new series of designs will also be listed, along with multidimensional extensions.

For convenience, we let $\binom{n}{m} = 0$ for m negative. The number of varieties, number of blocks, and block size are denoted by v, b and k, respectively.

2. Construction of designs based on line graphs.

In a design based on $L(G)$ each variety can be thought of as an edge of G. A block of size k is then a subgraph of G containing k edges. Therefore the design is simply a collection of subgraphs of G. In view of this, the following proposition is fairly obvious:

Proposition 2.1. Let $\Gamma = \{B_i\}_{i=1}^{b}$ be a collection of subgraphs of G, each consisting of k edges. Then Γ defines a block design based on $L(G)$ if the following two conditions are satisfied:

(i) Any two adjacent edges of G appear together in λ_1 subgraphs in Γ.

(ii) Any two nonadjacent edges of G appear together in λ_2 subgraphs in Γ.

So the question is how to choose a collection of subgraphs satisfying conditions (i) and (ii) of Proposition 2.1.

What we need is a collection of subgraphs which is "symmetric" enough in a sense yet to be defined. This suggests that we could start with some initial subgraph and then "symmetrize" it. We shall illustrate this idea on triangular and L_2 type PBIB designs.

Suppose π is a permutation of m vertices. For an edge e connecting vertex i and vertex j, we define $\pi(e)$ to be the edge connecting vertex $\pi(i)$ and vertex $\pi(j)$. If G is a subgraph of K_m consisting of t edges $e_1, e_2, \ldots, e_t$, then $\pi(G)$ is defined to be the subgraph of K_m consisting of the t edges $\pi(e_1), \pi(e_2), \ldots, \pi(e_t)$. A collection $\Gamma = \{G_i\}$ of subgraphs of K_m is said to be invariant under π if $\pi(G_i) \in \Gamma$ for all $G_i \in \Gamma$.

Under the above definitions, by Proposition 2.1, the following two propositions (which are our main results) are apparent.

<u>Proposition 2.2</u>. Let $\Gamma=\{B_i\}_{i=1}^{b}$ be a collection of subgraphs of K_n, each containing k edges. If Γ is invariant under all permutations of the n vertices, then Γ defines a triangular PBIB design.

<u>Proposition 2.3</u>. Let $\Gamma = \{B_i\}_{i=1}^{b}$ be a collection of subgraphs of $K_{n,n}$ each containing k edges, and let $A = \{a_1, a_2, \ldots, a_n\}$ and $B = \{b_1, b_2, \ldots, b_n\}$ be the two partite sets of $K_{n,n}$. Then Γ defines an L_2 type PBIB design if Γ is invariant under each of the following per-

mutations of the $2n$ vertices $a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n$:

(i) all permutations such that $\pi(A) = A$ and $\pi(B) = B$,

(ii) the permutation such that $\pi(a_i) = b_i$ and $\pi(b_i) = a_i$ for all $i = 1, 2, \ldots, n$.

Propositions 2.2 and 2.3 provide a very simple and general method for constructing triangular and L_2 type PBIB designs. One can start with <u>any</u> initial block and then "symmetrize" it to achieve the appropriate invariance, i.e., to find an invariant set of subgraphs containing the initial one. Since the choice of the initial block is arbitrary, our method can be used to construct triangular and L_2 type PBIB designs of any given block size.

3. <u>Applications and relations to earlier results in the literature</u>.

In this section, we shall demonstrate how various well-known methods for constructing triangular and L_2 type PBIB designs in the literature follow from our general method. This also illustrates how our method can be used in practice. Any user aware of the general principle can easily construct a desgin on his own.

Let us consider the construction of triangular designs first. Let the n vertices of K_n be $a_1, a_2, \ldots, a_n$, and the $n \times n$ square array defining a triangular scheme be denoted by T.

(i) Suppose we choose the initial block B_1 to be the complete graph on the m vertices $a_1, a_2, \ldots, a_m$, where $m < n$ is a positive integer. Then obviously the smallest Γ which contains B_1 and is invariant under all permutations

of $a_1, a_2, \ldots, a_n$ is the collection of all complete subgraphs of K_n on m vertices. Referring to the $n \times n$ square array T, we see that the initial block B_1 corresponds to the set of the varieties appearing in the first m rows as well as in the first m columns of T. The symmetrization amounts to the considersation of all choices of m rows and the m corresponding columns. This produces a triangular PBIB design with $v = \binom{n}{2}$, $b = \binom{n}{m}$, $k = \binom{m}{2}$, $r = \binom{n-2}{m-2}$, $\lambda_1 = \binom{n-3}{m-3}$ and $\lambda_2 = \binom{n-4}{m-4}$ for all $2 \le m \le n$ and all $n \ge 4$. When $m = 3$, it reduces to the following family constructed by Clatworthy (1956):

$$v = \binom{n}{2}, \; b = \binom{n}{3}, \; k = 3, \; r = n-2, \; \lambda_1 = 1, \; \lambda_2 = 0 .$$

(ii) For any positive integer $m < n$, let B_1 be the complete $(m, n-m)$-bipartite graph with $\{a_1, \ldots, a_m\}$ and $\{a_{m+1}, \ldots, a_n\}$ as the two partite sets. Then the smallest Γ which contains B_1 and is invariant under all permutations of $a_1, a_2, \ldots, a_n$ consists of all complete $(m, n-m)$-bipartite subgraphs of K_n. The resulting triangular design can be described as follows. Define a block to be the set of varieties which appear in m fixed rows, but <u>not</u> in the m corresponding columns of T. When all choices of m rows are considered, we obtain a triangular design with parameters

$$v = \binom{n}{2}, \; b = \binom{n}{m}, \; k = m(n-m), \; r = 2\binom{n-2}{n-m-1},$$

$$\lambda_1 = \binom{n-3}{n-m-1} + \binom{n-3}{m-1} = \binom{n-2}{m-1}, \; \lambda_2 = 4\binom{n-4}{m-2} .$$

This family is new. Note that when $m = 1$, it reduces to the following family

$$v = \binom{n}{2}, \quad b = n, \quad k = n-1, \quad r = 2, \quad \lambda_1 = 1, \quad \lambda_2 = 0,$$

which appeared in Shrikhande (1952) and Bose and Shimamoto (1952).

(iii) The family constructed in (ii) is in fact a subfamily of an even more general family which we now construct. For any two positive integers m_1, m_2 such that $m_1 + m_2 \leq n$, let B_1 be the complete (m_1, m_2)-bipartite graph with $\{a_1, a_2, \ldots, a_{m_1}\}$, and $\{a_{m_1+1}, \ldots, a_{m_1+m_2}\}$ as the two partite sets. Then the smallest Γ which contains B_1 and is invariant under all the permutations of $\{a_1, \ldots, a_n\}$ consists of all complete (m_1, m_2)-bipartite subgraphs of K_n. The resulting design again can easily be described in terms of the $n \times n$ square array T. A block is defined to be the set of varieties that appear in m_1 fixed rows and in m_2 fixed columns other than those corresponding to the m_1 rows. When all possibilities are considered, we end up with a triangular design with the following parameters:

$$v = \binom{n}{2}, \quad b = \binom{n}{m_1}\binom{n-m_1}{m_2}, \quad k = m_1 m_2, \quad r = \binom{n-2}{m_1-1}\binom{n-m_1-1}{m_2-1} +$$

$$+ \binom{n-2}{m_2-1}\binom{n-m_1-2}{m_1-1}$$

$$\lambda_1 = \binom{n-3}{m_1-1}\binom{n-m_1-2}{m_2-2} + \binom{n-3}{m_2-1}\binom{n-m_2-2}{m_1-2} \ , \ \lambda_2 = 2\left\{\binom{n-4}{m_1-2}\binom{n-m_1-2}{m_2-2} + \binom{n-4}{m_2-2}\binom{n-m_2-2}{m_1-2}\right\} .$$

When $m_2 = n-m_1$, this reduces to the family of designs constructed in (ii). Furthermore, if we let $m_1 = 1$, $m_2 = n-2$, then we obtain the following family

$$v = \binom{n}{2} \ , \ b = n(n-1) \ , \ k = n-2 \ , \ r = 2(n-2) \ , \ \lambda_1 = n-3 \ , \ \lambda_2 = 0 \ ,$$

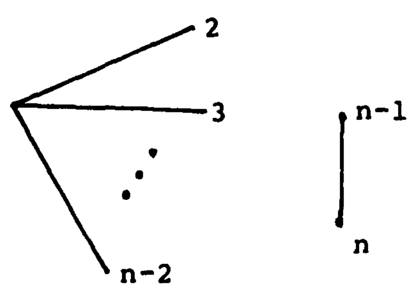
which appeared in Chang, Liu and Liu (1965), and Masuyama (1965).

(iv) Masuyama (1965) constructed a family of triangular designs with parameters

$$v = \binom{n}{2} \ , \ b = n\binom{n-1}{2}, \ k = n-2 \ , \ r = (n-2)^2 \ ,$$

$$\lambda_1 = \binom{n-3}{2}, \ \lambda_2 = 4 \ , \ n \geq 5 \ .$$

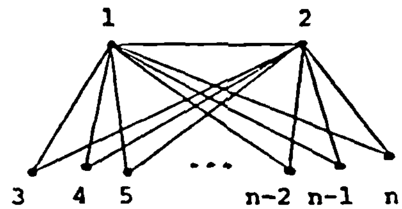
Blocks of size $k = n-2$ are obtained by taking $n-3$ treatments lying in the same row (or column) of T and one treatment that lies neither in the same row nor in the same column as the first $(n-3)$ treatments. Such a design is obtained by symmetrizing the following graph:



(v) Liu (1963) constructed a family of triangular PBIB designs with the following parameters:

$$v = \binom{n}{2} \ , \ b = \binom{n}{2} \ , \ r = k = 2(n-2) \ , \ \lambda_1 = n-2 \ , \ \lambda_2 = 4 \ , \ n \geq 5 \ .$$

To each variety there corresponds a block which consists of all its first associates. Such a design is obtained by symmetrizing the following graph:



Both families in (iv) and (v) can be extended to more general families. Other families can be constructed by choosing different initial blocks, but we won't pursue this here. The main purpose of this paper is to present and illustrate a general method. Once the method is understood, practitioners can construct designs on their own. For this reason, we shall not list any other new triangular design.

Now we turn to the construction of L_2 type PBIB designs. For convenience, we use Π to denote the set of all the permutations on $2n$ vertices $\{a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n\}$ satisfying conditions (i) and (ii) of Proposition 2.3 and let the $n \times n$ square array defining an L_2 scheme be denoted by S .

Let B_1 be the complete (m_1, m_2)-bipartite graph with $\{a_1, a_2, \ldots, a_{m_1}\}$ and $\{b_1, b_2, \ldots, b_{m_2}\}$ as the two partite sets, where $m_1 \le n$ and $m_2 \le n$ are two positive integers and at least one of them is $< n$. Then the smallest Γ which contains B_1 and is invariant under Π consists of all the complete (m_1, m_2)-bipartite subgraphs of $K_{n,n}$. The resulting L_2 type PBIB design can be described as follows. There are two types of blocks. One type of block consists of all the varieties appearing in m_1 rows and m_2 columns. The other type of block consists of all the varieties appearing in m_2 rows and m_1 columns. (Of course, there is no such distinction when $m_1 = m_2$). Considering all possible combinations, we obtain a family of L_2 type PBIB designs with the following parameters:

$$v = n^2, \quad b = 2\binom{n}{m_1}\binom{n}{m_2}, \quad k = m_1 m_2, \quad r = 2\binom{n-1}{m_1-1}\binom{n-1}{m_2-1},$$

$$\lambda_1 = \binom{n-1}{m_1-1}\binom{n-2}{m_2-2} + \binom{n-1}{m_2-1}\binom{n-2}{m_1-2}, \quad \lambda_2 = 2\binom{n-2}{m_1-2}\binom{n-2}{m_2-2}, \text{ for}$$

$$m_1 \ne m_2, \text{ and } v = n^2, \quad b = \binom{n}{m}^2, \quad k = m^2, \quad r = \binom{n-1}{m-1}^2,$$

$$\lambda_1 = \binom{n-1}{m-1}\binom{n-2}{m-2}, \quad \lambda_2 = \binom{n-2}{m-2}^2, \text{ for } m_1 = m_2 = m.$$

This family is new. Various subfamilies have appeared in the literature:

(a) Let $m_1 = 1$, then we have the following family of L_2 type PBIB designs:

$$v = n^2 \ , \ b = 2n\binom{n}{m_2} \ , \ k = m_2 \ , \ r = 2\binom{n-1}{m_2-1} \ , \ \lambda_1 = \binom{n-2}{m_2-2} \ , \ \lambda_2 = 0 \ ,$$

which has been constructed by Clatworthy (1967).

(b) Let $m_1 = 2$, $m_2 = n$, then we have a family of designs with the following parameters:

$$v = n^2 \ , \ b = 2\binom{n}{2} \ , \ k = 2n \ , \ r = 2(n-1) \ , \ \lambda_1 = n \ , \ \lambda_2 = 2 \ ,$$

which also appeared in Clatworthy (1967).

We shall stop our discussion and listing here. Our list of designs does not exhaust the new or old designs that can be constructed by this method. Any choice of initial block leads to a design once the right "symmetry" is formulated. This simple idea is also applicable to some other types of designs, e.g., $L_i(n)$ designs with $i > 2$.

Multidimensional extensions lead to families of PBIB designs with generalized triangular and L_2 schemes and more than two associate classes. In the case of the generalized triangular scheme, given n points, identify a variety by a subset of s points $(s \le n)$. We hence obtain $\binom{n}{s}$ varieties. By considering an initial block of k varieties $(k < \binom{n}{s})$ and its images under all possible permutations of the n points, we obtain a triangular PBIB design with at most s associate classes. As an example, if the initial block consists of all the subsets of s points of a fixed subset of m points $(s < m < n)$, then the images of this initial block under the $n!$ permutations of the n points leads to a PBIB

design with the following parameters: $v = \binom{n}{s}$, $b = \binom{n}{m}$, $k = \binom{m}{s}$, $r = \binom{n-s}{m-s}$ and $\lambda_i = \binom{n-s-i}{m-s-i}$, $1 \le i \le s$. It is easily seen that a design as above has a generalized triangular scheme with two varieties (as subsets of s points) being i^{th} associates if they differ in exactly i points. For example, if $n = 6$, $s = 3$ and the points are 1,2,3,4,5,6 , then the 20 varieties are lexicographically labeled $1 = \{123\}$, $2 = \{124\}$,..., $20 = \{456\}$. Fix 4 points, say 1,2,3 and 4 . The initial block consists then of varieties $1 = \{123\}$, $2 = \{124\}$, $5 = \{134\}$, $11 = \{234\}$. By "symmetrizing" this block under the 6! permutations of the points 1,2,3,4,5,6 we obtain

1	1	1	2	2	3	5	5	6	8	11	11	12	14	17
2	3	4	3	4	4	6	7	7	9	12	13	13	15	18
5	6	7	8	9	10	8	9	10	10	14	15	16	16	19
11	12	13	14	15	16	17	18	19	20	17	18	19	20	20

a PBIB design with 3 associate classes and parameters:

$$v = 20 \ , \ b = 15 \ , \ k = 4 \ , \ r = 3 \ , \ \lambda_1 = 1 \ , \ \lambda_2 = 0 \ , \ \lambda_3 = 0 \ .$$

By working with the multipartite graph on sn points, consisting of s partite sets of size n each, one can generate PBIB designs with up to s associate classes. To see this, label a variety by an ordered s-tuple whose i^{th} entry is a point in the i^{th} partite set. We generate n^s

varieties by doing so. Select any block of k varieties. The images of this block under all the $s!(n!)^s$ permutations of the sn points which preserve the s partite sets give a PBIB design with s asssociate classes. As an illustration, select m $(s < m < n)$ points in each partite set. Consider the m^s varieties whose components are among the points selected. If we think of these m^s varieties as a block, then the procedure described above gives a PBIB design with $v = n^s$, $b = \binom{n}{m}^s$, $k = m^s$, $r = \binom{n-1}{m-1}^s$ and $\lambda_i = \binom{n-i}{m-i}^{s-i}\binom{n-2}{m-2}^i$, $1 \le i \le s$. A more convenient way of thinking about this example is to visualize the m^s varieties as points of a hypercube with s dimensions. Choose independently m components along each one of the s coordinates. A block is defined as the collection of the m^s varieties whose coordinates belong to the chosen set of m components along each coordinate. When all the $\binom{n}{m}$ choices of components are independently considered along each coordinate, we obtain a PBIB design with parameters as mentioned. Two varieties are i^{th} associates if they differ in exactly i coordinates as points of the hypercube.

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